Analysis, Representation and Compression of ECG signals

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WDA_2009
The ECG signal:

- it is a time-varying signal;
- analysis of the local morphology of the ECG and its time varying properties is an important clinical diagnostic tool;

ECG signals

- are associated with electrical changes known as depolarization and repolarization of the atria and ventricles.
ECG signals

- need to be stored and compressed without significant loss of signal quality;
- this requires reliable and adequate methods;
- the P wave in the ECG signal is a result of the contraction of the atria;
- the contraction of the ventricles produces the QRS complex;
- its repolarization manifests itself in the T wave;
- the repolarization of the atria is hidden by the dominant QRS morphology.
In signal processing:

- the **Fourier analysis** is one of the most widely spread tools where:
  - the signal is represented by its trigonometric Fourier transform;

- **Lately:** regarding the approximation and compression of signals new type of representations appeared:
  - approx. by splines
- the **Wavelet transform**:  

- the representation of signals by **RATIONAL FUNCTIONS** which is successfully used in many areas like **Control Theory**;

**our results**:  

- the method can be successfully used for the representation, analysis and compression of ECG signals as well;


5. Levente Lócsi: Approximating poles of complex rational functions
to be published: Acta Univ. Sapientiae, Mathematica (accepted)
- **Our goal:**
  - the representation and compression- without significant loss of an ECG signal;

- **In Control Theory:**
  - the approximation by rational functions is used to determine the transfer function of the system;
  - the number of parameters needed is **CONSIDERABLY LESS THEN BY OTHER METHODS.**
- For the approximation of the ECG signals we will use rational functions having their poles outside the unit circle;

- First:
  - we have to determine the poles of the rational functions used for the approximation
  - the multiplicity of the poles
  - the coefficients of the linear combination.
- In the present work
  - we will deal by periodic functions only, which are defined on the unit circle of the open disc $D$:

  \[ D := \{ z \in \mathbb{C} : |z| < 1 \} \]

  - the periodic signals are considered continuous functions on the unit circle $T$:

  \[ T := \{ z \in \mathbb{C} : |z| = 1 \} \]
These functions will be approximated by complex rational functions having their poles outside the unit circle.

The mirror of the pole, with respect to T;
the approximation by rational functions is done in two steps:

- **first** we determine the poles of the elementary rational waves used in the approximation and their multiplicity;

- **then**, choosing the most frequently used basis, the Malmquist-Takenaka system: we determine the coefficients of the linear combination in the space spanned by these basis;
- The approximation of the poles of complex rational functions it is not a trivial task.

- Levente Lócsi adapted the Nelder-Mead simplex method to the case when: the function is defined by its values on the unit circle.

Levente Lócsi:

*Approximating poles of complex rational functions*

*Acta Univ. Sapientiae, Mathematica* (acc. for publ.)
- This method allows us to fix in advance the number of poles used in the approximation process, together with their multiplicity.

- With these parameters we introduce a finite dimensional function space consisting of elementary rational waves, generated by the poles in question;
- we fix a sequence:

\[ a_n \in D \ (n = 1, 2, \ldots) \]

and define the multiplicity of

\[ a_n \] by \[ m_n = \sum_{a_i = a_n, \ i \leq n} 1. \]

- we consider the set of fun.: \( \Theta := (\phi_k : k = 1 \ldots m) \subset A \)

with

\[ \phi_k(z) = \frac{1}{(1 - a_k z)^{m_k}} \]
- applying the Gram-Schmidt orthogonalization procedure to the elementary function system \( \Theta \)
- we get

**The Malmquist-Takenaka system**

- introducing the Blaschke-functions:

\[
B_b(z) := \frac{z - b}{1 - b z} \quad (b \in D, z \in C)
\]

- where:

\[
z = e^{it} ; \quad b^* := \frac{1}{b};
\]
- the MT system can be expressed in a simple analytical form:

\[ \Phi_n(z) := \frac{\sqrt{1-|a_n|^2}}{1-a_n z} \prod_{k=1}^{n-1} B_{a_k}(z) \quad (z \in C, n = 2, 3, \ldots), \]

with: \[ \Phi_1(z) := \frac{\sqrt{1-|a_1|^2}}{1-a_1 z}. \]
- The Malmquist-Takenaka (MT) functions form an orthonormal system on the unit circle $T$, i.e.:

$$\langle \Phi_n, \Phi_m \rangle := \frac{1}{2\pi} \int_0^{2\pi} \Phi_n(e^{it})\Phi_m(e^{it})dt = \delta_{nm} \quad (m, n \in \{1, 2, \ldots\}),$$

where $\delta_{nm}$ is the Kronecker symbol.

- The continuous functions on the unit circle, form an Euclidian space with respect to this scalar product.
- the function, ECG signal, is then appr. by the partial sum of the expansion in Fourier series with respect to this basis;

- the rational function used for the appr. is the linear comb. of the real and imaginary part of the rational waves det. by the poles and their multiplicity;

- The partial sums of the Fourier expansion with respect to the MT system gives the best approximation:

  in the norm of the Euclidean space.
- Given a function \( f \), in our case an ECG signal, we compute the orthogonal projection of the function on the subspace \( \phi \),

- and look for the best approximation of the function in the norm induced by the scalar product in span \( \phi \);
- let us denote by: 
\[ P_\phi f = P_{a_1, \ldots, a_m} f \]
the orthogonal projection of \( f \) on the subspace \( \text{span } \phi \) given by:
\[ P_\phi f = \sum_{n=1}^{m} \langle f, \phi_n \rangle \phi_n. \]

- if we denote by: 
\[ \varepsilon_\phi f = \varepsilon_{a_1, \ldots, a_m} f \]
the difference between the function to be approximated, and the best approximation in \( \| \cdot \|_2 \),
\[ \mathcal{E}_\phi f = \left\| f - P\phi f \right\|_2 \]

- then this means that we have to determine that sequence: \( a_n \in D(n = 1, 2, \ldots, m) \)

- that is, those poles together with their multiplicity, which minimizes \( \mathcal{E}_\phi f \).

- that is, that \( P\phi f \) which gives the best approximation of the function \( f \).
- as can be seen, the function approximating best, depends on the poles we have determined previously.

- the Nelder-Mead algorithm, with which we appr. poles stops, when the standard deviation of the $\mathcal{E}_\phi f$ values in the vertices of the simplex becomes less then a predefined value;
Steps:
- fix the nr. of poles together with their multiplicity;
- use the Nelder-Mead alg. to appr. poles;
- minimize $\varepsilon_\phi f$.

- repeat until...

NR. OF DATA USED FOR THE APPR...
- on the next slides you can see some of our results and the effect of varying the number of poles as well as their multiplicity;

100_1 MIT-BIH

- bad choice:
  3 poles of multipl. 2

multipl.3
ST-depressio - ischemia

Izoelektromos vonal - isometric line

Dr. Laszlo Hejjel MD.,
Med. Heart Inst. Pecs
3 poles of multipl. 3
4 poles mult. 3
4 poles mult. 3
100_2 p3m3
p4m3
p4m4 – no need – in this case

-before 3. QRS
p4m3
Kamrai ES (ventricular extrasystole)
Ang: premature beat

Dr. Laszlo Hejjel MD.,
Med. Heart Inst. Pecs
Kedves Ildiko,

Jo az ekg-rekonstrukcio, egy-egy elvezetesbol nehez neha megmondani a diagnozist, az alapvonal is elegge vandorol, de par ekg-t visszakuldtem feliratozva.

Angolul: ventricular premature beat, ST segment elevation/depression, baseline-wandering

Az ST szakasz változásai ischaemiara (elevatio - kezdődi infarctusra) utalnak általában.

Erezze jól Magat és hasznos idotoltest!

Udv.: L.
PRD: 9.63

21.600 - 2.313
Kamrai ES (ventricular extrasystole)
Ang: premature beat

Dr. Laszlo Hejjel MD.,
Med. Heart Inst. Pecs
ST-elevatio -
Sign of the beginning infarct

Dr. Laszlo Hejjel MD.,
Med. Heart Inst. Pecs

ST-"depressio", de inkább ST elevatio tükörképe a szemközti területnek megfelelően

- or better, the mirror image of ST elevation corr. to the opp. site.
p4m4
“depessio”, de inkább ST elevatio tükörképe a szemközti területnek megfelelően

- or better, the mirror image of ST elevation corr. to the opp. site.
- the decision has to be...
- the decision has to be...
109_2_p3m3

109_2_p4m3 — better
p4m5
no need...

p4m4
Köszönöm SZÍVES figyelmük!

Isten éltesse Tanár Urak!!!
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And now...